# Written Exam for the B.Sc. in Economics autumn 2012-2013 

## Microeconomics C

Final Exam

February 20
(2-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students’ self-service system.

## PLEASE ANSWER ALL QUESTIONS.

## PLEASE EXPLAIN YOUR ANSWERS.

1. (a) Consider the normal-form game below. Do iterated elimination of strictly dominated strategies (explain briefly each step). Which strategies survive?

|  | $E$ | $F$ | $G$ | $H$ |
| :---: | :---: | :---: | :---: | :---: |
| $A$ | 3,3 | 6,4 | 5,7 | 5,8 |
| $B$ | 2,5 | 6,1 | 3,8 | 4,9 |
| $C$ | 1,6 | 8,7 | 6,2 | 6,1 |
| $D$ | 7,1 | 7,2 | 6,7 | 5,1 |

(b) Consider the following normal-form game ( $x$ is a constant):

|  | $L$ | $M$ | $R$ |
| :--- | :--- | :--- | :--- |
| $U$ | 4,3 | 2,1 | $5, x$ |
| $D$ | 1,1 | 3,2 | $2,4 x$ |

Find all values of $x$ such that $R$ is a strictly dominated strategy for player 2. For these values of $x$, find all (pure and mixed) Nash equilibria in the game.
2. Consider the following two stage game with two players (1 and 2). In the first stage, player 1 chooses either $S$ (for stop) or $G$ (for go). If he chooses $S$, the game ends and each player receives a payoff of 3. If he chooses $G$, the game proceeds to stage two. In stage two, the two players play the following simultaneous game:

Player 1

|  | Player 2 |  |  |
| :--- | :--- | :--- | :--- |
|  | $L$ | $M$ | $R$ |
| $U$ | 0,0 | 5,1 | 3,0 |
| $D$ | 1,3 | 2,2 | 4,2 |

(a) Draw a game tree representing the two stage game. Is it a game of perfect or imperfect information? Write down the set of strategies for each player.
(b) Find all pure strategy subgame perfect Nash equilibria.
(c) Find all pure strategy Nash equilibria. Are they all subgame perfect? Explain.
3. Two musicians $(i=1,2)$ are working on a joint song. The effort musician $i$ puts into writing the song is $e_{i} \geq 0$. The final quality of the song is determined by the effort choices of the musicians in the following way:

$$
Q\left(e_{1}, e_{2}\right)=2\left(e_{1}+e_{2}\right)-A e_{1} e_{2},
$$

where $A>0$ is a constant. The cost of effort for the musicians are:

$$
C_{i}\left(e_{i}\right)=\left(e_{i}\right)^{2} \text { for each } i=1,2 .
$$

The utility for each musician is equal to the quality of the song minus his cost of effort:

$$
U_{i}\left(e_{1}, e_{2}\right)=Q\left(e_{1}, e_{2}\right)-C_{i}\left(e_{i}\right) \text { for each } i=1,2
$$

(a) Consider the game where the musicians choose their effort levels simultaneously and independently. Find the Nash equilibrium $\left(e_{1}^{*}, e_{2}^{*}\right)$. How do the effort levels depend on $A$ ? Give an intuitive explanation.
(b) Find the social optimum $\left(e_{1}^{S O}, e_{2}^{S O}\right)$, i.e., the effort levels that maximize total utility.
(c) Let $A=1$. Suppose the game studied in (a) is repeated over an infinite time horizon $t=1,2, \ldots, \infty$. The discount factor of each musician is $\delta \in(0,1)$. In this infinitely repeated game, specify trigger strategies such that the outcome of each stage is $\left(e_{1}^{S O}, e_{2}^{S O}\right)$. Find the inequality that must be satisfied for the trigger strategies to constitute a subgame perfect Nash equilibrium. Find the lowest value of $\delta$ such that the inequality is satisfied.
4. (a) Find a pooling perfect Bayesian equilibrium in the signaling game below:

(b) Two friends, Antonio and Tommy, are bargaining over how to share the 600 -gram cake that Antonio's grandmother has baked for them. Antonio's utility from consuming $x_{A}$ grams of cake is

$$
u_{A}\left(x_{A}\right)=3 x_{A} .
$$

Tommys utility from consuming $x_{T}$ grams of cake is

$$
u_{T}\left(x_{T}\right)=x_{T} .
$$

If they fail to reach an agreement, Antonio's grandmother will give the cake to one of her neighbors, so that neither of the two friends receives anything.
Represent the situation as a bargaining problem, i.e., draw the sets $X$ and $U$ and mark the disagreement points. Find the Nash bargaining solution.

